been born in Cork, and who published his work on the natural history of Spain in Paris in 1776. It would be very interesting to trace any conclusions as to extinct volcanoes that were current in Bowles's mind as early as 1750. Ozy is not likely to have made a show of his previous knowledge to men of established position like Guettard and Malesherbes, who had honoured the local apothecary with a call. He listened to their exposition of the craters, and "avoua ingénument" that he was much surprised at what he heard. After all, it was Guettard who took the matter up absolutely fresh from the beginning, and whose memoir made it for the first time public knowledge.

Bowles may have come from Catalonia, and may have formed his opinions there. I make no mention of Olzendorff, of whom I have no further trace; but the fact that Bowles instructed Ozy at Clermont, if the above contention is correct, a year before Guettard had formed his conclusions, and in an age rife with unfounded speculation, marks him as a geological observer deserving of more credit than he has yet received. I called attention to Ozy's letter in *Knowledge* for 1898, p. 266, and have since made inquiries through friends in Cork and elsewhere; but the present family of Bowles in Cork, formerly Boles, can furnish no data as to the life of the mineralogist.

An interesting inquiry also arises as to when the Giant's Causeway in Antrim was first regarded as a lava-flow. What did Bowles know of this phenomenon? Its detailed appreciation, from a geological point of view, is usually ascribed to Whitehurst, in the second edition of his work in 1786. But Faujas de St. Fond in 1778 calls many of the French lava-flows "chaussées," and clearly shows his own conclusions when he styles certain examples with good columnar jointing "pavés des géans."

GRENVILLE A. J. COLE.

Royal College of Science for Ireland, Dublin, March 1.

## Probability—James Bernoulli's Theorem.

IT may possibly be of some little interest to notice that the theorem in probability, which goes by the name of James Bernoulli's theorem, alluded to in my letter to NATURE of December 13, 1900 (p. 154), admits of a treatment somewhat more elementary than the usual one.

The theorem may be stated thus:—If $p$ is the probability of a given event, and $n$ the number of times considered; as $n$ increases without limit, the probability that the ratio of the number of times in which the event happens to the whole number of times ($n$) will only deviate from $p$ within limits of excess and defect, which decrease indefinitely as $n$ increases without limit, is a probability which approaches indefinitely to unity as its limit.

In Laplace's demonstration (see Todhunter's "History of Probability," art. 993) Stirling's theorem, for the evaluation of factorials, is used in the first step; in the second step the theorem of Euler,

$$\Sigma y = \int y\,dr - \tfrac{1}{2}y + \tfrac{1}{12}\frac{dy}{dr} - \ldots$$

which is also implied in the usual proof of Stirling's theorem; and, finally, the result depends on the evaluation of the well-known definite integral $\int_0^\tau e^{-t^2}dt$.

Further, it is essential to this demonstration to make the limit of deviation in excess from the ratio $p$ equal to the limit of deviation in defect, for then as members of the series, which represent the probability sought, equidistant from the middle of the series contain certain terms equal in magnitude and of contrary sign, these terms cancel in the addition of such pairs, and are thus got rid of.

It may be shown that the general result of Bernoulli's theorem may be got without the above described use of Euler's theorem (*i.e.* the second use of it), without using the evaluation of $\int_0^\tau e^{-t^2}dt$, and without making the limits of excess and defect equal. These limits may have any ratio whatever.

Let $q$ be the probability that the event does not happen, so that $p+q=1$.

Let the whole number of times considered be $y+x$. Since this is to increase without limit, we may suppose $p(x+y)$ and $q(x+y)$ always integers.

Let P be the probability that the number of the times in which the event happens be between $p(x+y)+ax$ and $p(x+y)-bx$ where $(a+b)=1$, so that $x$ represents (so to speak) the range of the variation, $a$ and $b$ may have any ratio to one another. Assume that $y=mx^{2(1-\kappa)}$, where $\kappa$ may be as small as we please, but finite. Thus P is the probability that the ratio of the times when the event happens to the whole number of times shall not exceed $p$ by more than $\frac{ax}{x+y}$, or fall short of $p$ by more than $\frac{bx}{x+y}$; limits which vanish when $x$ and $y$ are infinite.

Let $P_1=$ probability that the number of times in which the event happens is less than $p(x+y)-bx$, and $P_2$ the probability that it exceeds $p(x+y)+ax$. Then $1-P=P_1+P_2$.

$$P_1=p^{x+y}+(x+y)p^{x+y-1}q+\ldots + \frac{(x+y)!\;p^{p(x+y)+ax+1}q^{q(x+y)-ax-1}}{\{p(x+y)+ax+1\}!\;\{q(x+y)-ax-1\}!}.$$

Now $P_2$ evidently = the probability that the number of cases in which the event does not happen is less than $q(x+y)-ax$, and therefore the series for $P_2$ is derivable from that for $P_1$, by interchanging $p$ and $q$, and by interchanging $a$ and $b$. These values of $P_1$ and $P_2$ may, of course, be also got from the equation

$$P_1+P_2=(p+q)^n-P.$$

$P_1$ is evidently less than the geometrical progression of which the sum is

$$\frac{(x+y)!\;p^{p(x+y)+ax}q^{q(x+y)-ax}}{\{p(x+y)+ax\}!\;\{q(x+y)-ax\}!}\cdot\frac{\left\{\frac{p}{q}\cdot\frac{q(x+y)-ax}{p(x+y)+ax+1}\right\}^{q(x+y)-ax+1}-1}{\frac{p}{q}\,\frac{q(x+y)-ax}{p(x+y)+ax+1}-1}$$

By Stirling's theorem, $x$ and $y$ increasing *ad. inf.*

$$(x+y)! = (x+y)^{x+y+\frac{1}{2}}e^{-(x+y)}\sqrt{2\pi}\left(1+\frac{1}{12(x+y)}+\ldots\right)$$
$$= y^{x+y+\frac{1}{2}}\left(1+\frac{x}{y}\right)^{x+y+\frac{1}{2}}e^{-(x+y)}\sqrt{2\pi}\left(1+\frac{1}{12(x+y)}+\ldots\right),$$

and similarly for the other factorials.

Thus the above expression becomes

$$\frac{1}{\sqrt{2\pi pq}}\;\frac{\left(\frac{1}{12(x+y)}+\ldots\right)}{\left(1+\frac{1}{12(p(x+y)+ax)}+\ldots\right)\left(1+\frac{1}{12(q(x+y)-ax)}+\ldots\right)}.$$
$$\frac{\left(1+\frac{x}{y}\right)^{x+y+\frac{1}{2}}}{\left(1+\frac{(a+p)x}{py}\right)^{p(x+y)+ax+\frac{1}{2}}\left(1+\frac{(q-a)x}{qy}\right)^{q(x+y)-ax+\frac{1}{2}}}\cdot\frac{1}{y^{\frac{1}{2}}}.$$
$$\frac{\left\{\frac{p}{q}\,\frac{q(x+y)-ax}{p(x+y)+ax+1}\right\}^{q(x+y)-ax+1}-1}{\frac{p}{q}\,\frac{q(x+y)-ax}{p(x+y)+ax+1}-1}.$$

The limit of the second factor is unity. The third factor may be shown to become in the limit

$$\frac{1}{\frac{a^2}{e^{2mpq}}}\quad x^{2\kappa}.$$

The limit of

$$\left\{\frac{p}{q}\,\frac{q(x+y)-ax}{p(x+y)+ax+1}\right\}^{q(x+y)-ax+1}$$

is the limit of $e^{-cx}$, and the limit of

$$y^{\frac{1}{2}}\left(\frac{p}{q}\cdot\frac{q(x+y)-ax}{p(x+y)+ax+1}-1\right)=-hx^{\kappa},$$

where $c$ and $h$ are positive constants. Hence the limit of the product of the factors is zero—that is, the limit of $P_1$ is zero, and evidently also the limit of $P_2$.

Hence the limit of P is unity.

The range of the deviation ($x$) is greater in this proof than in the usual one, for in the latter $x$ would vary as $y^{\frac{1}{2}}$ as against $y^{\frac{1}{2-2\kappa}}$ where $\kappa$ may be as small as we please.

A further simplification would be introduced by a method of evaluating the series $P_1$ or $P_2$ without the use of Stirling's theorem. Such a method has been given me by Mr. G. G. Berry, of Balliol College, and may be briefly described as follows:—

If in the expansion of $(p+q)^n$ we stop $t$ terms before the greatest, the truncated series has a smaller sum than the G.P., which has the same two final terms. If $t^2$ is great as compared with $n$, the G.P. has a sum which vanishes compared with its final term multiplied by $\sqrt{n}$. But the product of the greatest term of $(p+q)^n$ and $\sqrt{n}$ is finite; for the sum of $\sqrt{n}$ terms on either side of the greatest $<1$, and the ratio of the greatest term to a term distant from it by $\sqrt{n}$ places is—

$$\frac{\left(1-\frac{1}{pn}\right)\left(1-\frac{2}{pn}\right)\cdots\left(1-\frac{\sqrt{n}-1}{pn}\right)}{\left(1+\frac{1}{qn}\right)\left(1+\frac{2}{qn}\right)\cdots\left(1+\frac{\sqrt{n}}{qn}\right)},$$

which has a finite limit. J. COOK WILSON.

Oxford.

### A Tree Torn by Lightning.

I ENCLOSE two photographs of an oak tree struck by lightning, which seem of interest.

The storm, one of considerable violence, occurred on July 27, 1900, and continued for several hours. The tree stood by the side of a road which runs over the Chilterns from Ipsden, a little village about five miles from Wallingford and ten from Henley. It was standing at the western edge of a small stretch of woodland. The opposite side of the road was quite clear and sloped down to the plain.

FIG. 1.

On examination, the bark was found to be completely stripped off and flung on one side; a large branch was torn away, and the fractured end was extraordinarily splintered and smashed. So far as I saw there were no signs of charring.

The inner surface of the bark was marked longitudinally with thin wavy lines, very close-set, of which the crests were about ¼ inch apart.

The first photograph gives a general view of the tree; the second represents the lower side of the bent portion of the trunk, and shows very clearly the rending effect of the lightning on a fibrous tissue.

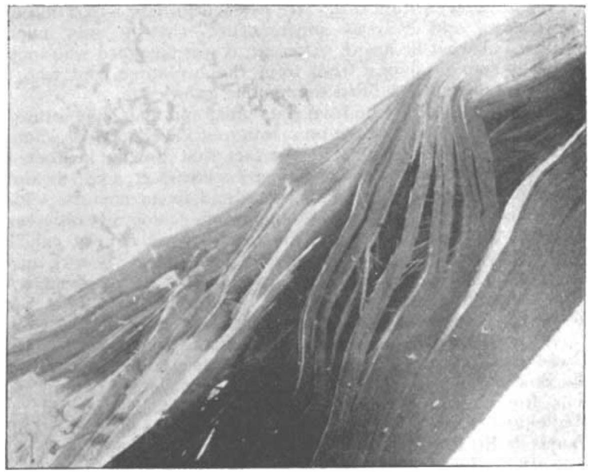

FIG. 2.

The photographs were taken about a fortnight after the tree was struck, during which time there had been much wind and rain. PERCY E. SPIELMANN.

### Adaptation of Instinct in a Trap-door Spider.

THE following extract from the *Sydney Bulletin*, January 12, sent to me by a correspondent in Western Australia, recounts an observation sufficiently interesting, I think, to be reprinted and put on permanent record in a scientific periodical:—"A friend of mine noticed near his camp a trap-door spider run in front of him and pop into its hole, pulling the 'lid' down as it disappeared. The lid seemed so neat and perfect a circle that the man stooped to examine it, and found, to his astonishment, that it was a sixpence! There was nothing but silk thread covering the top of the coin, but underneath mud and silk thread were coated on and shaped convex (as usual). The coin had probably been swept out of the tent with rubbish."

As is well known, the doors of trap-door spiders' burrows are typically made of flattened pellets of earth stuck together with silk or other adhesive material. The unique behaviour of the spider in question showed no little discrimination on her part touching the suitability as to size, shape and weight of the object selected to fulfil the purpose for which the sixpence was used. R. I. POCOCK.

March 6.

### Protective Markings in Cats.

IT will probably appear to many—as it does to myself—that the development of a protective mechanism in a domestic animal is not likely, and for several reasons—such as the shortness of time at the disposal of the race, and, of course, to their large independence of stress of circumstances. Still it may be admitted that the domestic cat bears its subjugation to man more lightly than many of the other creatures which he has tamed. The particular mark above the eye to which your correspondent refers (p. 441) has also been pointed out by Mr. Wallace in the dog. It may interest those of your readers who are not aware of the fact to learn that the tiger has a largish and very bright white spot upon the back of the ear. When the ears are directed forwards this spot is exceedingly conspicuous from in front (as any one may verify upon the fine pair of tigers now in these gardens); and, in the dimness of a cave or a thicket, might conceivably produce an impression of alertness when the animal was really sleeping.

Zoological Society's Gardens. FRANK E. BEDDARD.